

Big-Bang Cosmology with Photon Creation

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The temperature evolution law is determined for an expanding FRW type Universe with a mixture of matter and radiation where “adiabatic” creation of photons has taken place. Taking into account this photon creation we discuss the physical conditions for having a hot big bang Universe. We also compare our results to the ones obtained from the standard FRW model.

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I. INTRODUCTION

It is widely believed that matter and radiation need to be created in order to overcome some conceptual problems of the standard hot big-bang cosmology [1]. The most popular approach accounting for the phenomenon of creation is based on the idea that the early Universe underwent an inflationary phase during which the temperature decreased nearly 10^{28} orders of magnitude. At the end of this supercooling process, the energy density of the inflaton field was completely or almost completely converted into radiation, and the resulting Universe could have been reheated in less than one expansion Hubble time [2]. However, there are theories where the gravitational particle creation phenomenon is conceived with no appealing for inflation and, consequently, allow the creation process to occur continuously in the course of the evolution. Probably, the best example is the adiabatic vacuum mechanism invented long ago by Parker and collaborators using the Bugoliubov mode-mixing technique in the context of quantum field theory in curved spacetimes [3,4]. However, this approach is plagued with several conceptual and mathematical difficulties. In particular, there is not a well-defined prescription of how the created matter and/or radiation should be incorporated in the Einstein field equations (EFE) [5].

More recently, a new phenomenological macroscopic approach to gravitational creation of matter and radiation has attracted considerable attention [6]- [16]. In this framework, the creation event of the inflationary scenario is also replaced by a continuous creation process. The crucial ingredients of this formulation are a balance equation for the number density of the created particles and a negative pressure term in the stress tensor so that the back-reaction problem present in Parker's mechanism is naturally avoided. Another advantage of this formulation is that the laws of non-equilibrium thermodynamics were used since the very beginning, thereby leading to definite relations among the classical thermodynamic quantities. In particular, the creation pressure depends on the creation rate in a well defined form, and potentially may alter significantly several predictions of the standard big-bang cosmology. Completing such an approach, a spectrum for blackbody radiation when photon creation takes place has also been proposed in the literature [16,17]. This spectrum is preserved during a free expansion (for instance, after decoupling between matter and radiation), and more important still, it is compatible with the present spectral shape of the cosmic background radiation (CBR).

On the other hand, in the photon-conserving Friedmann-Robertson-Walker (FRW) Universes, the temperature of the matter content follows the radiation temperature law when there is any thermal contact between these components. This state of affairs define what is called a hot big-bang Universe. Usually, the condition that the Universe underwent

a very hot phase in its beginning is expressed by requiring that there are many photons for each proton or neutron in the Universe today. This fact allows one to establish the cosmic eras, and is closely related to the high value of the radiation specific entropy (per baryon) in the present Universe.

In this letter, by taking into account the photon creation process described by the thermodynamic formulation of irreversible processes, we analyze the temperature evolution law for the matter-energy content in the framework of a FRW metric. Our aim here is to discuss under which conditions the basic concept of hot big bang Universe remains valid when a continuous photon creation phenomenon is considered.

II. CBR SPECTRUM AND THE TEMPERATURE LAW WITH PHOTON CREATION

Let us consider a spectrum of photons whose number and energy densities are, respectively, $n_r \sim T^3$ and $\rho_r \sim T^4$ and let $N_r(t)$ be the instantaneous comoving total number of photons, where T is the temperature. Since $N_r = n_r R^3$, where $R(t)$ is the scale factor of a FRW cosmology, one may write

$$N_r(t)^{-\frac{1}{3}} T R = \text{const} \quad . \quad (1)$$

It thus follows that if $N_r(t)$ is a constant the temperature law of the photon conserving FRW model, $TR = \text{const}$, is recovered. From the scale factor-redshift relation, $R = R_o(1+z)^{-1}$, the above temperature law becomes

$$T = T_o(1+z)\left(\frac{N_r(t)}{N_{or}}\right)^{\frac{1}{3}} \quad , \quad (2)$$

where the subscript o denotes the present day value of a quantity. As we shall see this relation has some interesting physical consequences, and a consistent cosmological framework with photon creation may be traced back using this new temperature law.

As shown by one of us [16,17], equation (1) leads to the following spectral distribution:

$$\rho_T(\nu) = \left(\frac{N_r(t)}{N_{or}}\right)^{\frac{4}{3}} \frac{8\pi h}{c^3} \frac{\nu^3}{\exp\left[\left(\frac{N_r(t)}{N_{or}}\right)^{\frac{1}{3}} \frac{h\nu}{kT}\right] - 1} \quad . \quad (3)$$

In the absence of creation ($N_r(t) = N_{or}$), the standard Planckian spectrum is recovered [18]. The derivation of the above spectrum depends only on the new temperature law and satisfies the equilibrium relations

$$n_r(T) = \int_0^\infty \frac{\rho_T(\nu) d\nu}{\left(\frac{N_r(t)}{N_{or}}\right)^{\frac{1}{3}} h\nu} = bT^3 \quad , \quad (4)$$

and

$$\rho_r(T) = \int_0^\infty \rho_T(\nu) d\nu = aT^4 \quad , \quad (5)$$

where $b = \frac{0.244}{h^3 c^3}$ and $a = \frac{\pi^2 k^4}{15 h^3 c^3}$, are the blackbody radiation constants. A gravitational photon creation process satisfying the above equilibrium relations has been termed “adiabatic” creation [8,16].

The temperature law (1) implies that the exponential factor appearing in the spectrum given by Eq.(3) is time independent. As a consequence, the spectrum is not destroyed as the Universe evolves, at least not after the transition from an opaque to a transparent Universe. Note also that the above distribution cannot be distinguished from the blackbody spectrum at the present epoch when $T = T_o$ and $N_r(t_o) = N_{or}$. In what follows we study under which conditions the temperature law (1) may be applied before decoupling, that is, during the time when matter and radiation were in thermal contact.

Let us now consider a mixture of a non-relativistic gas in thermal contact with the blackbody radiation described by Eq.(3). For completeness we set up the basic equations including “adiabatic” creation of both components.

For this system, the total pressure (p) and energy density (ρ) are given by ($c = 1$)

$$\rho = nm + (\gamma - 1)^{-1} nkT + aT^4 = \rho_m + \rho_r \quad , \quad (6)$$

$$p = nkT + \frac{1}{3}aT^4 + p_{rc} + p_{mc} \quad , \quad (7)$$

where n is the number density of gas particles, m is their mass, γ is the specific ratio of the gas (5/3 for a monoatomic gas). The quantities p_{rc} and p_{mc} stand for radiation and matter creation pressures, respectively. In the “adiabatic” formulation they assume the following form [8,9]:

$$p_{rc} = -\frac{\rho_r + p_r}{3n_r H} \psi_r \quad , \quad (8)$$

and

$$p_{mc} = -\frac{\rho_m + p_m}{3n H} \psi_m \quad , \quad (9)$$

where $H = \dot{R}/R$ is the Hubble parameter and ψ_r , ψ_m denote, respectively, the creation rates of radiation and matter components. The number densities, n and n_r , satisfy the balance equations:

$$\frac{\dot{n}}{n} + 3\frac{\dot{R}}{R} = \frac{\psi_m}{n} \quad , \quad (10)$$

and

$$\frac{\dot{n}_r}{n_r} + 3\frac{\dot{R}}{R} = \frac{\psi_r}{n_r} \quad . \quad (11)$$

In the context of the FRW metric, the energy conservation law reads [19]

$$\frac{d}{dR}(\rho R^3) = -3pR^2 \quad . \quad (12)$$

Before proceeding further, it is worth noticing that the analysis of the temperature evolution law may be separated in several cases: (i) the standard model ($\psi_r = \psi_m = 0$); (ii) photon creation ($\psi_r \neq 0, \psi_m = 0$); (iii) matter creation ($\psi_r = 0, \psi_m \neq 0$); and (iv) radiation and matter creation ($\psi_r \neq 0, \psi_m \neq 0$). In this letter we are primarily interested in the case of photon creation whose spectrum is defined by Eq.(3). Thus, henceforth we restrict our attention to the case (ii), for which $\psi_m = p_{mc} = 0$.

Inserting Eq.(6) and Eq.(7) into Eq.(12) it follows that

$$\frac{1}{R^2} \frac{d}{dR} [nmR^3 + (\gamma - 1)^{-1}nkTR^3 + aT^4R^3] = -3nkT - aT^4 + \frac{4}{3}aT^4 \frac{\psi_r}{n_r H} \quad . \quad (13)$$

Now, by considering that the number of massive particles is conserved, one obtains from Eq.(10)

$$\frac{d}{dR}(nR^3) = 0 \quad . \quad (14)$$

Thus, substituting Eq.(14) into Eq.(13), and dividing the result by $3nkT$, we get

$$\frac{R}{T} \frac{dT}{dR} = -\frac{1 + \sigma_r - \sigma_r \beta}{\frac{1}{3}(\gamma - 1)^{-1} + \sigma_r} \quad , \quad (15)$$

where the parameters σ_r and β are defined by

$$\sigma_r = \frac{4aT^3}{3nk} \quad , \quad (16)$$

and

$$\beta = \frac{\psi_r}{3n_r H} \quad . \quad (17)$$

The quantity σ_r is the specific radiation entropy (per gas particle) while β quantifies how relevant is the photon creation process in an expanding Universe. These parameters are dimensionless, and in general, are also time-dependent quantities. For $\beta = 0$, equation (15) reduces to that one of the standard model [19]. It is worth noticing that the β parameter may be rewritten as $\beta = \frac{\Gamma}{H}$, where $\Gamma = \frac{\psi_r}{3n_r}$. Thus, β is the ratio between the “interaction rate” of the creation process and the expansion rate of the Universe. Naturally, Γ or equivalently ψ_r , must be determined

from a kinetic theoretical approach or from a quantum field theory. In any case, a reasonable upper limit to this rate is $\Gamma = H$, since for this value the photon creation rate exactly compensates for the dilution of particles due to expansion (see Eq.(11)). A new cosmological scenario will be thus obtained only if $0 < \beta \leq 1$. In particular, for a radiation dominated model ($p = \frac{1}{3}\rho$), the limiting case $\beta = 1$ is a radiation-filled de Sitter Universe. Naturally, if β is small, say, if $\Gamma \simeq 10^{-3}H$ or smaller, for all practical purposes the photon creation process may be safely neglected.

Returning to equation (15) we see that in the limit $\sigma_r \gg 1$, the temperature evolution equation reduces to

$$\frac{R}{T} \frac{dT}{dR} = -(1 - \beta) \quad , \quad (18)$$

or still, using Eq.(11) and Eq.(17)

$$\frac{R}{T} \frac{dT}{dR} = \frac{\dot{n}_r}{3n_r H} \quad . \quad (19)$$

This equation can be rewritten as

$$\frac{dT}{T} = \frac{dN_r}{3N_r} - \frac{dR}{R} \quad , \quad (20)$$

where $N_r(t)$ is the instantaneous comoving number of photons. Integrating the above expression we obtain:

$$N_r(t)^{-\frac{1}{3}} T R = \text{const} \quad , \quad (21)$$

which is the same temperature law for a freely propagating blackbody spectrum with photon creation (see Eq.(1)).

Therefore, as long as the quantity σ_r is large, the radiative component will continue to overpower the material component. Hence, while there is any significant thermal contact between them, the matter temperature will follow Eq.(21) as well. This means that the condition for a hot big bang cosmology is not modified when “adiabatic” photon creation occurs. Note that the above result holds regardless of the value of the β parameter and assures the validity of the above equation during a considerable part of the evolution of the Universe. The important point here is that the cosmic eras are still viable using the above generalized temperature law. However, unlike in the standard model, the radiation specific entropy does not remain constant when photon creation takes place. Since $n_r \propto T^3$ the usual expression

$$\sigma_r = 0.37 \frac{n_r}{n} = 0.37 \frac{N_r(t)}{N} \quad (22)$$

remains valid nonetheless n_r does not vary proportionally to R^{-3} . As a consequence, the variation rate of σ_r is directly proportional to the variation rate of $N_r(t)$, which in turn depends on the magnitude of the β parameter. So, if β approaches to zero, $N_r(t)$ and σ_r assume their constant values, and the standard model results are recovered.

III. A SPECIFIC MODEL

We consider the simplest photon creation model for which the parameter β is constant. This scenario can be defined by taking into account the “interaction” rate $\Gamma = \alpha H$, where α is a positive constant smaller than unity. As one may check from Eq.(18), in this case the temperature scaling law assumes the simple form

$$T \propto R^{-(1-\alpha)} \quad , \quad (23)$$

or still, in terms of the redshift

$$T = T_o(1+z)^{1-\alpha} \quad , \quad (24)$$

so that for $z > 0$ the Universe is cooler than the standard model.

Considering the balance equation written as

$$\dot{n}_r + 3(1-\alpha)n_r H = 0 \quad ,$$

we obtain,

$$n_r \propto R^{-3(1-\alpha)} \quad , \quad (25)$$

which could be obtained directly from the number density-temperature relation (see Eq.(4)). As a consistency check, by substituting $N_r = n_r R^3$ in the expression above, it is easily seen that the temperature law also follows directly from the generalized expression Eq.(21). Replacing Eq.(25) into Eq.(22) one may see that

$$\sigma_r = \sigma_{or} \left(\frac{R}{R_o} \right)^{3\alpha} \quad , \quad (26)$$

or still, in terms of the redshift

$$\sigma_r = \frac{\sigma_{or}}{(1+z)^{3\alpha}} \quad , \quad (27)$$

where $\sigma_{or} \sim 10^{8-9}$ is the now observed radiation specific entropy.

The above equations are a concrete example that the usual physical conditions defining a hot big bang cosmology may be weakened. In particular, specific entropy so large (and constant) as 10^8 is not required, providing that the product TR varies in the course of the evolution. In other words, 10^8 is only the present value of an increasing time-dependent quantity. Therefore, instead of predicting the currently observed value, the important question in this

framework is how a reasonable “initial value”, say $\sigma_r \simeq 10^2$, could be explained from the first principles. In particular, for the toy model presented here, it follows from equations (24) and (27), that a value of $\sigma_r \simeq 10^2$ in the beginning of the nucleosynthesis epoch is possible only if $\alpha \simeq 0.18$. More details on the nucleosynthesis of light elements, using a properly modified nucleosynthesis code which considers “adiabatic” creation of neutrinos and effectively massless species at nucleosynthesis epoch, will be discussed in a forthcoming communication [21].

In conclusion, we have considered an evolutionary Universe where “adiabatic” photon creation has taken place. The conditions defining a big bang scenario with a blackbody spectrum endowed with photon creation and compatible with the present observed CBR distribution have been discussed. As we know, earlier approaches to matter creation processes, for instance, the steady state model, C-field theory, scale-covariant theory and others [22], fail the test of the CBR spectrum. As we have seen, this does not happen with the thermodynamic approach considered here. Naturally, in order to have a viable alternative to the photon-conserving FRW model, other cosmological properties need to be investigated. In particular, it would be important to use the Sachs-Wolf effect to test a big bang model with “adiabatic” photon creation.

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